M.Sc. Paper IX Mathematics Sem II

Theorem Prove that the property of a space being a T_0-space is a topological property

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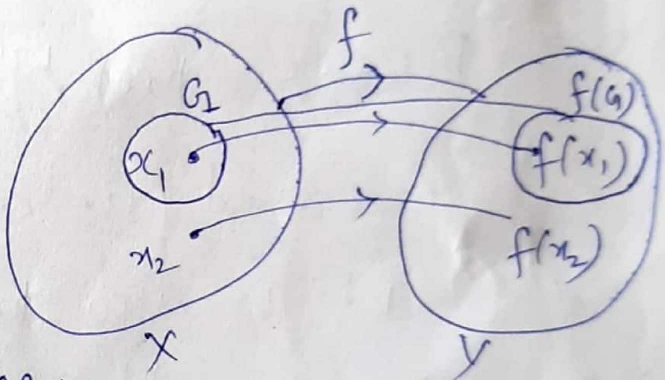
or

Prove that the property of a space being a T_0-space is preserved under one-one, onto and open mapping

or

If $f: X \to Y$ be one-one, onto and open, and X be a T_0-space then Y is also a T_0-space.

Proof



Let (X, T_1) be a T_0-space.

Let $f: X \to Y$ where (Y, T_2) is a topo. space.

Let f be one-one, onto and open.

To Prove $\to$ (Y, T_2) is a T_0-space.

Let the points $y_1, y_2 \in Y$ and $y_1 \neq y_2$.

$\because$ f is one-one, onto $\Rightarrow \exists$ two points x_1 and x_2, $(x_1 \neq x_2)$ s.t. $f(x_1) = y_1$, $f(x_2) = y_2$.

Since (X, T_1) is a T_0-space, by definition of T_0-space, $\exists$ a T_1-open set

G such that $x_1 \in G$ but $x_2 \notin G$.

$\because$ f is open $\Rightarrow$ f(G) is also an open set

and f(G) contains x_1 but not x_2.

Thus, in (Y, T_2), $\exists$ an open set f(G)

such that f(G) contains the point $f(x_1) =$

but not $f(x_2) = y_2$.

By definition of T_0-space, we can say that

$\Rightarrow$ (Y, T_2) is a T_0-space.

Hence it is a topological property.

Because property of being T_0-space is

preserved under one-one onto mapping.

It is also preserved under

homeomorphism. Hence, its' a topological

property.

Theorem: Prove that every subspace of a T_0-space is a T_0-space.

Soln

Let (X, T) be a T_0-space.

Let (Y, T_1) be a subspace of (X, T)

where T_1 is T-relative topology.

Let y_1, y_2 be two points of Y

i.e. points $y_1, y_2 \in Y$ and $y_1 \neq y_2$.

$\because Y \subset X \Rightarrow y_1, y_2 \in X$ and $y_1 \neq y_2$

Since (X, T) is a T_0-space, by definition

$\exists$ a T-open set G such that

$y_1 \in G$ but $y_2 \notin G$.

Also, by definition, $G \cap Y$ is a T_1-open set in which

$y_1 \in G \cap Y$ but $y_2 \notin G \cap Y$.

$\Rightarrow (Y, T_1)$ is also a T_0-space.

T_1 - Space or FRECHET Space

Let (X, T) be a topological space.

Let the points $x, y \in X$ and $x \neq y$

(i.e. the distinct points x, y are in X).

Then (X, T) is called a T_1-space iff

$\exists$ two open sets G and H such that

$x \in G$ but $y \notin G$

and $y \in H$ but $x \notin H$.

$\boxed{\text{Imp}^{t}}$ Every T_1-space is a T_0-space.

Proof: Let (X, T) be a T_1-space.

So, by definition, for every distinct pairs of points x and y, $\exists$ two open sets G and H s.t

$x \in G$ but $y \notin G$

and $y \in H$ but $x \notin H$.

$\Rightarrow$ $\exists$ an open set G such that

$x \in G$ but $y \notin G$.

$\Rightarrow$ (X, T) is a T_0-space (by definition of T_0-space).